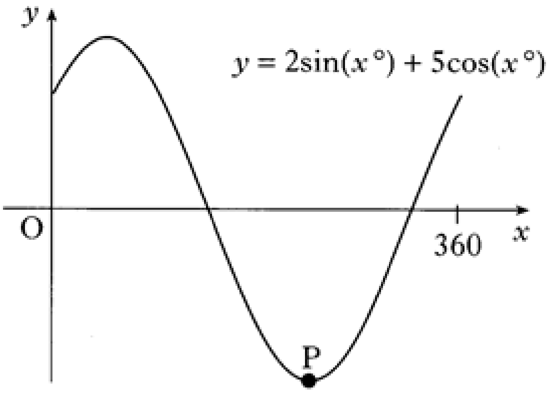
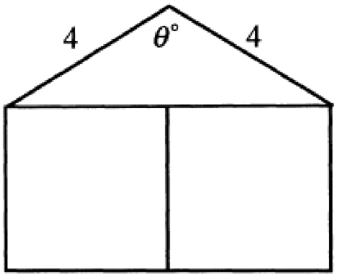
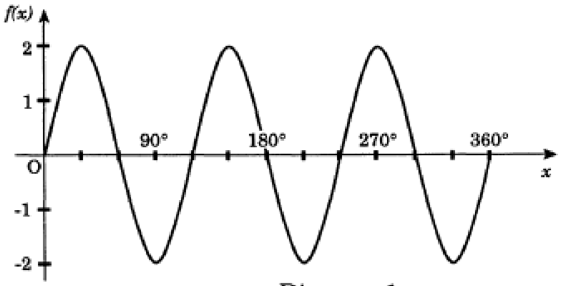
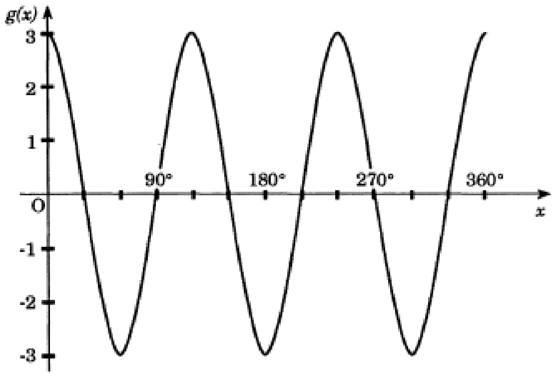
Wave Function (Degrees)

2001 P2	**5.** Express $8\cos x° - 6\sin x°$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 < a < 360$. **4**
3. (JAN) 02 P2	(*a*) Write $\sqrt{3}\sin x° + \cos x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a < 360$. **4** (*b*) Hence find the maximum value of $5 + \sqrt{3}\sin x° + \cos x°$ and determine the corresponding value of x in the interval $0 \leq x \leq 360$. **2**
2003 P2	**7.** Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.  (*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \leq a < 360$. **4** (*b*) Find the coordinates of the minimum turning point P. **3**
2004 P2	**6.** (*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. **4** (*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \leq x \leq 90$. **3**
2008 SP P2	**3.** (*a*) Express $5\sin x° - 12\cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 < a < 360$. **4** (*b*) Hence solve the equation $5\sin x° - 12\cos x° = 6{\cdot}5$ in the interval $0 < x < 360$. **3**
2008 SP2 P2	**4.** (*a*) Express $3\cos x° + 5\sin x°$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. **4** (*b*) Hence solve the equation $3\cos x° + 5\sin x° = 4$ for $0 \leq x \leq 90$. **3**
2010 P2	**2.** (*a*) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \leq a < 360$. Calculate the values of k and a. **4** (*b*) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$. (ii) Determine the values of x, in the interval $0 \leq x < 360$, at which these maximum and minimum values occur. **3**
2017 P1	**14.** (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x - a)°$, where $k > 0$ and $0 < a < 360$. **4** (b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \leq x \leq 360$. **3**

Pre 2000 - Wave Function (Degrees)

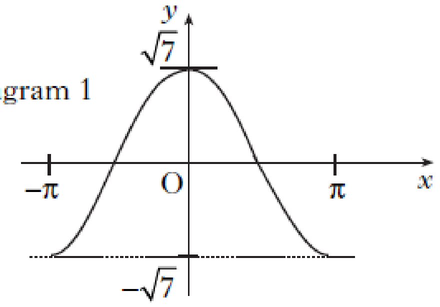
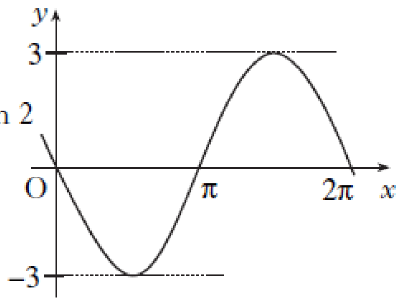
1	Express $2\sin x° - 5\cos x°$ in the form $k\sin(x-\alpha)°$, $0 \le \alpha < 360$ and $k > 0$. **4**
2	Solve the simultaneous equations $k\sin x° = 5$ $k\cos x° = 2$ where $k \ge 0$ and $0 \le x \le 360$ **4**
3	(*a*) Express $\sin x° - 3\cos x°$ in the form $k\sin(x-a)°$ where $k > 0$ and $0 \le a < 360$. Find the values of k and a. (*b*) Find the maximum value of $5 + \sin x° - 3\cos x°$ and state a value of x for which this maximum occurs. **4, 2**
4	(*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k \sin(x - a)°$, where $k > 0$ and $0 \le a < 360$. Calculate the values of k and a. **4** (*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \le x < 360$. **2**
5	$f(x) = 2\cos x° + 3\sin x°$. (*a*) Express $f(x)$ in the form $k\cos(x-\alpha)°$ where $k > 0$ and $0 \le \alpha < 360$. **(4)** (*b*) Hence solve algebraically $f(x) = 0.5$ for $0 \le x < 360$. **(3)**
6	Solve the equation $2\sin x° - 3\cos x° = 2\cdot 5$ in the interval $0 \le x < 360$. **8**
7	(*a*) Express $3\sin x° - \cos x°$ in the form $k\sin(x-\alpha)°$, where $k > 0$ and $0 \le \alpha \le 90$. **(4)** (*b*) Hence find algebraically the values of x between 0 and 180 for which $3\sin x° - \cos x° = \sqrt{5}$. **(4)** (*c*) Find the range of values of x between 0 and 180 for which $3\sin x° - \cos x° \le \sqrt{5}$. **(2)**
8	The function f is defined by $f(x) = 2\cos x° - 3\sin x°$. (*a*) Show that $f(x)$ can be expressed in the form $f(x) = k\cos(x+\alpha)°$ where $k > 0$ and $0 \le \alpha < 360$, and determine the values of k and α. **(4)** (*b*) Hence find the maximum and minimum values of $f(x)$ and the values of x at which they occur, where x lies in the interval $0 \le x < 360$. **(4)** (*c*) Write down the minimum value of $(f(x))^2$. **(1)**
9	(*a*) Show that $2\cos(x+30)° - \sin x°$ can be written as $\sqrt{3}\cos x° - 2\sin x°$. **(3)** (*b*) Express $\sqrt{3}\cos x° - 2\sin x°$ in the form $k\cos(x+\alpha)°$ where $k > 0$ and $0 \le \alpha \le 360$ and find the values of k and α. **(4)** (*c*) Hence, or otherwise, solve the equation $2\cos(x+30)° = \sin x° + 1$, $0 \le x \le 360$. **(3)**

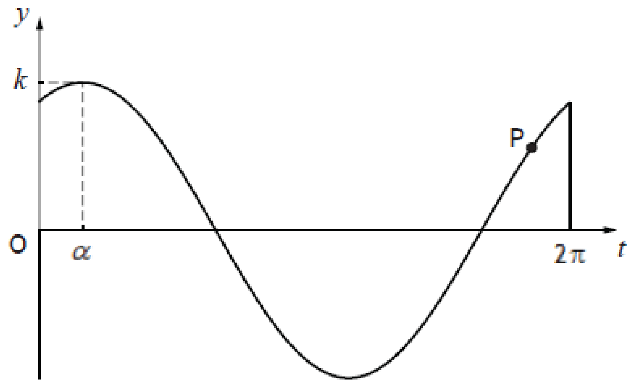
10	A builder has obtained a large supply of 4 metre rafters. He wishes to use them to build some holiday chalets. The planning department insists that the gable end of each chalet should be in the form of an isosceles triangle surmounting two squares, as shown in the diagram.  (*a*) If $\theta°$ is the angle shown in the diagram and A is the area (in square metres) of the gable end, show that $A = 8\left(2 + \sin\theta° - 2\cos\theta°\right)$. **(5)** (*b*) Express $8\sin\theta° - 16\cos\theta°$ in the form $k\sin(\theta - \alpha)°$. **(4)** (*c*) Find algebraically the value of θ for which the area of the gable end is 30 square metres. **(4)**
11	The displacement, d units, of a wave after t seconds, is given by the formula $d = \cos 20t° + \sqrt{3}\sin 20t°$. (*a*) Express d in the form $k\cos(20t - \alpha)°$, where $k > 0$ and $0 \le \alpha \le 360$. **(4)** (*b*) Sketch the graph of d for $0 \le t \le 18$. **(4)** (*c*) Find, correct to 1 decimal place, the values of t, $0 \le t \le 18$, for which the displacement is 1·5 units. **(3)**
12	The formula $d = 200 + 80(\cos 30t° + \sqrt{3}\sin 30t°)$ gives an approximation to the depth of water, d, measured in centimetres, in a harbour t hours after midnight. (*a*) Express $f(t) = \cos 30t° + \sqrt{3}\sin 30t°$ in the form $k\cos(30t - \alpha)°$ and state the values of k and α, where $0 \le \alpha \le 360$. **(4)** (*b*) (i) Use your result from part (*a*) to help you sketch the graph of $f(t)$ for $0 \le t \le 12$. (ii) Hence, on a separate diagram, sketch the graph of d for $0 \le d \le 12$. **(6)** (*c*) What is the "low-water" time at the harbour during the time interval shown on your graph? **(1)** (*d*) If the local fishing fleet needs at least 1·5 metres depth of water to enter the harbour without risk of running aground, between what hours must it avoid entering the harbour during the time interval shown on your graph? **(2)**

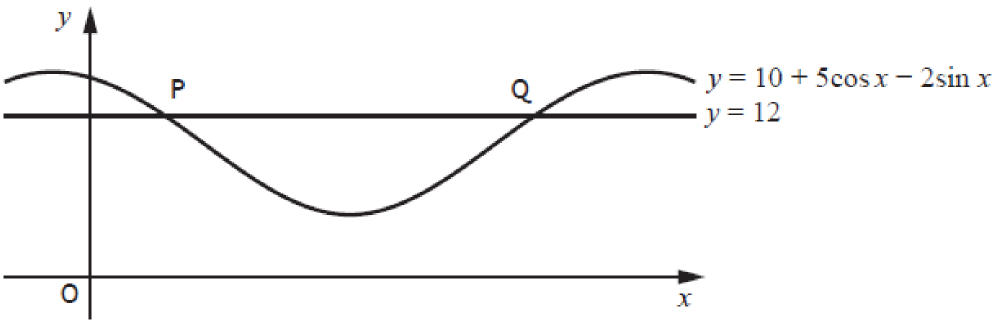
13	(*a*) (i) Diagram 1 shows part of the graph of the function f defined by $f(x) = b\sin ax^{\circ}$, where a and b are constants. Write down the values of a and b.  Diagram 1 (ii) Diagram 2 shows part of the graph of the function g defined by $g(x) = d\cos cx^{\circ}$, where c and d are constants. Write down the values of c and d.  Diagram 2 **(4)** (*b*) The function h is defined by $h(x) = f(x) + g(x)$. Show that $h(x)$ can be expressed in terms of a single trigonometric function of the form $q\sin(px + r)^{\circ}$ and find the values of p, q and r. **(5)**

Wave Function (Radians)

2000 P1	**B10.** Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$.	6
2002 P1	**9.** (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. (*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis.	4 3
2005 P1	**10.** (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq 2\pi$. (*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \leq x \leq 2\pi$.	4 5
2006 P2	**10.** A curve has equation $y = 7\sin x - 24\cos x$. (*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. (*b*) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1.	 4 3
2007 P1	**11.** (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. (*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$.	4 4

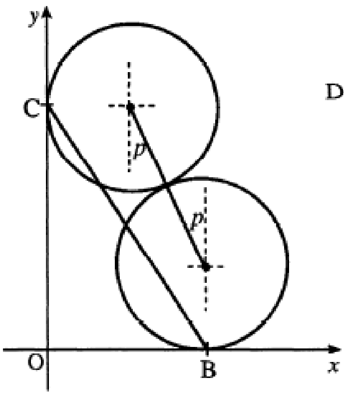
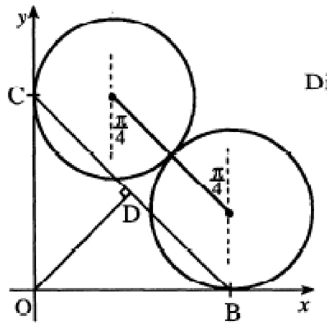
2008 P2	**3.** (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$,where $f(x) = p\cos x$. Write down the value of p. Diagram 1  (ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$. Write down the value of q. Diagram 2  	2
	(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$.	4
	(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression.	2
2012 P1	**22.** (*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos\left(x + a\right)$ where $k > 0$ and $0 \leq a < 2\pi$. Calculate the values of k and a.	4
	(*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \leq x \leq 2\pi$.	3
2013 P1	**23.** (*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k\ \sin(x - a)°$, where $k > 0$ and $0 \leq a < 360$. Calculate the values of k and a.	4
	(*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \leq x < 360$.	2
2015 OLD P2	**8.** The blades of a wind turbine are turning at a steady rate. The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula $h = 36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t) + 65.$ Express $36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t)$ in the form $k\sin(1{\cdot}5t - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$, and hence find the **two** values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn.	8

2015 SP P2	**10.** Two sound sources produce the waves $y = \sin t$ and $y = \sqrt{3}\cos t$. An investigation into the addition of these two waves produces the graph shown, with equation $y = k\cos(t-\alpha)$ for $0 \le t \le 2\pi$.  (a) Calculate the values of k and α. **4** The point P has a y-coordinate of $1{\cdot}2$. (c) Hence calculate the value of the t-coordinate of point P. **4**
2015 EP P1	**9.** The expression $\cos 4x - \sqrt{3}\sin 4x$ can be written in the form $k\cos(4x+a)$ where $k>0$ and $0 \le a \le 2\pi$. (a) Calculate the values of k and a. **4** (b) Find the points of intersection of the graph of $y = \cos 4x - \sqrt{3}\sin 4x$ with the x axis, in the interval $0 \le x \le \frac{\pi}{2}$. **3**
2015 P2	**9.** The blades of a wind turbine are turning at a steady rate. The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula $h = 36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t) + 65.$ Express $36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t)$ in the form $k\sin(1{\cdot}5t - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$, and hence find the two values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn. **8**

2016 P2	8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x+a)$, where $k > 0$ and $0 < a < 2\pi$. (b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$. The line cuts the curve at the points P and Q.  Find the x-coordinates of P and Q.	4 4

Pre 2000 - Wave Function (Radians)

1	(a) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x+a)$ where $k > 0$ and $0 \leq a < 2\pi$. Calculate the values of k and a. (b) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \leq x \leq 2\pi$.	4 3

2	Two identical coins, radius 1 unit, are supported by horizontal and vertical plates at B and C. Diagram 1 shows the coins touching each other and the line of centres is inclined at p radians to the vertical.  Diagram 1 Let d be the length of BC. (a) (i) Show that $\mathrm{OB} = 1 + 2\sin p$ **(1)** (ii) Write down a similar expression for OC and hence show that $d^2 = 6 + 4\cos p + 4\sin p$. **(2)** ($b$) (i) Express d^2 in the form $6 + k\cos(p - \alpha)$ **(4)** (ii) Hence write down the exact maximum value of d^2 and the value of p for which this occurs. **(2)** (c) Diagram 2 shows the special case where $p = \frac{\pi}{4}$.  Diagram 2 (i) Show that $\mathrm{OB} = 1 + \sqrt{2}$ and find the exact length of BD. **(2)** (ii) Using your answer to (b)(ii) find the exact value of $\sqrt{6 + 4\sqrt{2}}$. **(2)**